* Capacitance

* Poisson's and Laplace equations :-

$$\nabla \cdot \bar{D} = \rho_v$$

$$\bar{D} = \epsilon \bar{E}$$

$$E = -\nabla V$$

$$\nabla \cdot \epsilon \bar{E} = \rho_v$$

$$\epsilon \nabla \cdot (-\nabla V) = \rho_v$$

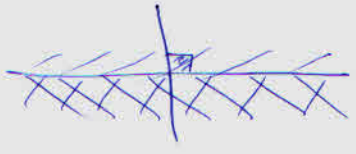
$$\nabla \cdot (\nabla V) = \frac{-\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \Rightarrow \text{Poisson's equation}$$

if $\rho_v = 0$ $\therefore$ $\boxed{\nabla^2 V = 0} \Rightarrow$ Laplace equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad (x, y, z)$$

1. Determination of $V = f(x, y, z, \rho_v)$
2. Determination of $\bar{E} = -\nabla V$
3. Determination of $\bar{D} = \epsilon \bar{E}$ $\quad \bar{a}_x, \bar{a}_y, \bar{a}_z$
4. From the normal component of $\bar{D}$ (wrt interface) we can get ρ_s C/m^2
5. Determination of $Q = \int_s \rho_s \, ds$



6. Determination of the capacitance $C = \frac{Q}{V}$

ex// Find the Capacitance of a Parallel Plate

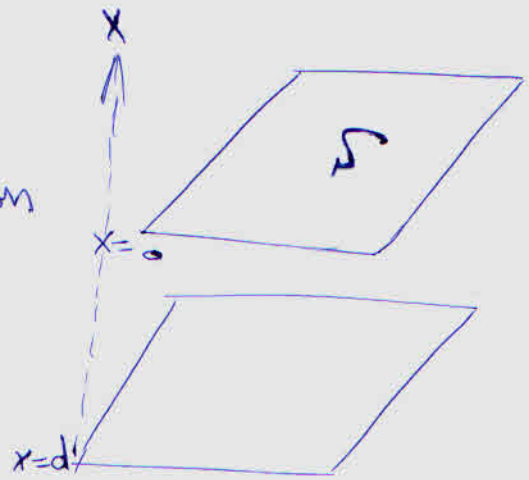
Capacitance of: 1- Plate area S

2- At $X=0$ & $V=0$

$X=d$ & $V=V_0$

Find the Capacitance of this Capacitor:

$$C = \frac{\epsilon S}{d} \quad F$$



using Laplace's equation

1- Determination of V

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

X لأنه يعتمد على ... سهل

$$\frac{\partial^2 V}{\partial X^2} = 0$$

$$V = AX + B$$

at $X=0 \Rightarrow V=0 \Rightarrow B=0$

$$X=d \Rightarrow V=V_0 = Ad \Rightarrow A = \frac{V_0}{d}$$

$$\boxed{V = \frac{V_0}{d} X}$$

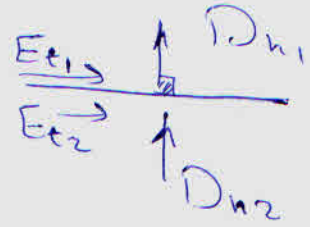
2- Determination of E

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial X}\bar{a}_X + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right]$$

$$= -\frac{\partial V}{\partial X}\bar{a}_X = -\frac{V_0}{d}\bar{a}_X$$

3- Determination of $\bar{D}$

$$\bar{D} = \epsilon \bar{E} = \frac{-\epsilon V_0}{d} \bar{a}_x$$



$E_{t_1} = E_{t_2}$

$D_{n_1} - D_{n_2} = \rho_s$

if medium ② is a conductor

$E_{t_2} = 0$

$D_{n_2} = 0$

4- Determination of ρ_s

$$D_N = D \bar{a}_x = \rho_s$$

$$\boxed{\rho_s = \frac{\epsilon V_0}{d}}$$ ← هيه ثابتة لا تعتمد على إتجاهات

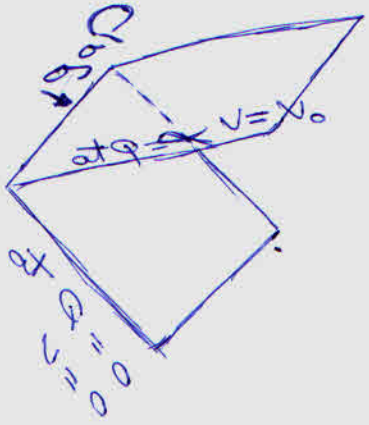
5- Determination of Q

$$Q = \frac{\epsilon V_0}{d} S$$

6- Determination of Capacitance

$$C = \frac{Q}{V_0} = \frac{\epsilon S}{d}$$

Ex For the capacitor shown in the Figure, find C using Laplace's equation



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